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# Generation of Gravitational Waves with Nuclear Reactions

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**Abstract.** The problem of efficient generation of High Frequency Gravitational Waves (HFGWs) and pulses of Gravitational Radiation might find a reasonably simple solution by employing nuclear matter, especially isomers. A fissioning isomer not only rotates at extremely high frequency (~ $3.03 \times 10^{24}$ $s^{-1}$), but is also highly deformed in the first stages of fission (the nucleus is rotating and made asymmetric "before" fission). Thus one achieves significant impulsive forces (e.g., $3.67 \times 10^{8}$ N) acting over extremely short time spans (e.g., $3.3 \times 10^{-22}$ s). Alternatively, a pulsed particle beam, which could include antimatter, could trigger nuclear reactions and build up a coherent GW as the particles move through a target mass. The usual difficulty with HFGWs generated by nuclear reactions is the small dimensions of their nuclear-reaction volumes, that is, the small moment of inertia and submicroscopic radii of gyration (e.g., $10^{-16}$ m) of the nuclear-mass *system*. Such a difficulty is overcome by utilizing clusters of nuclear material, whose nuclear reactions are in synchronization (through the use of a computer controlled logic system) and are at a large distance apart, e.g., meters, kilometers, etc. The effective radius of gyration of the overall nuclear mass system is enormous and if the quadrupole formalism holds even approximately, then significant HFGW is generated, for example up to $8.5 \times 10^{10}$ W to $1.64 \times 10^{25}$ W bursts for the transient asymmetrical spinning nucleus case. In this preliminary analysis, possible conceptual designs of reactors suitable for the generation of HFGWs are discussed as well as applications to space technology. In an optimized dual-beam design, GW amplitudes on the order of A ~ 0.005 are theoretically achieved in the laboratory, which might have interesting general-relativity and nuclear-physics consequences.

**Keywords:** high frequency, gravitational wave, HFGW, nuclear fission, nuclear reactor, isomer, particle accelerator.
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## INTRODUCTION

Since the 1960's a relatively small group of scientists have developed an interest in generators of High Frequency Gravitational Waves (HFGWs), for which a number of interesting practical applications have been discovered. It should be noted that Hawking and Israel (1979) have defined HFGWs as having frequencies in excess of 100 kHz. Also since the 1960's time period most of the research efforts in the world have been devoted to the detection of Low-Frequency Gravitational Waves (LFGWs) of astrophysical origin, because it is considered the best opportunity to directly discover this elusive radiation. In the seventies the situation was somewhat different and there was some interest in demonstrating the existence of Gravitational Waves (GWs) with a self consistent experiment and possibly with a controlled source.

One proposed GW source was the expanding neutron cloud from a nuclear fusion explosion (Chapline, Nuckolls and Wood, 1974). The cloud was modeled by an expanding cylinder of neutrons with constant density, continuously re-supplied by neutrons from the main axis. Although the energy involved was the highest our technology could manage at the time, the power emitted in GW was found to be so low as to be considered undetectable. On the other hand, detection was not the main concern. As in the case of this paper the main concern is the possible effect of GWs on matter and we will leave detection to a future analysis. The authors of the Chapline paper considered neutrons traveling at constant velocity in the expanding cylinder.

This is reasonable, but we know from the quadrupole equation (quadrupole formalism) for a spinning rod that the emission of GWs is proportional to the square of the "jerk, "shake"" or time derivative of the acceleration (impulse). Obviously this has a maximum shortly after the nucleus has been destabilized by the fission-inducing particle, a neutron or an antiproton as recently suggested (Baker, 2000). The nucleus destabilizes and the fission fragments depart propelled by electrical forces. More precisely, two main changes in the quadrupole moment are involved during fission: (1) the change of the distance of the fragments from the rotational axis and (2) the change in rotational speed due to conservation of angular momentum. The second is effective only in nuclei with large angular momentum and the combination of the two is the source of strong GWs. Many of such nuclear systems can be excited in sequence to superimpose the amplitude (and memory effect) of the emitted GW. The effective power is then proportional to the square of the number of the emitting systems in the sequence. If the speed of fission inducing particles has a very narrow distribution around a value equal to a fraction of the speed of light, then a plane wave-front of fission inducing particles can be sent towards a "sheet" of "fuel" particles at an angle to induce a "fission wave" traveling at the speed of light (i.e., the speed of GW) along the "sheet." This mechanism produces the main pulse of GW. It is also recognized, as mentioned above, that the quadrupole equation for the GW power produced by a spinning rod can be formulated in terms of a jerk or impulsive forces acting at the ends of a dumbbell-shaped rod of a given length. If we imagine a nucleus spinning down at each end of and comprising such a dumbbell, then this length could be quite large, e.g., meters, kilometers, or more, and the quadrupole moment greatly increased since its magnitudes increases with the square of the distance apart.

The usual difficulty with HFGWs generated by nuclear reactions is the small dimensions of their nuclear-reaction volumes, that is, the small moment of inertia and submicroscopic radii of gyration (e.g., $10^{-16}$ m) of the nuclear-mass *system*. Such a difficulty is overcome in the analyses of this paper by utilizing clusters of nuclear material (e.g., at the ends of the "dumbbell"), whose nuclear reactions are in synchronization (through the use of a computer controlled logic system) and are at a large distance apart, e.g., meters, kilometers, etc. The effective radius of gyration of the overall nuclear mass system is enormous and, if the quadrupole formalism even approximately holds, then significant HFGW is generated, for example up to $8.5\mathrm{x}10^{10}$ W to $1.64\mathrm{x}10^{25}$ W bursts for the transient asymmetrical spinning nucleus case. This situation is analogous to the double-pulsar system J0737-3039A and B observed by Lyne, *et al.* (2004), which is expected to generate significant Low-Frequency Gravitational Waves (LFGWs).

## THE QUADRUPOLE FORMULA AND NUCLEAR FISSION

By solving numerically the equations of a fissioning rotating nucleus, it is possible to discover that the rate of change of the quadrupole moment peaks and the quadrupole is a maximum when the nucleus is in a highly deformed state and before the two fragments start to depart in opposite directions (Bonasera, Kondratyev and Iwamoto, 1997), the rotational speed is only slightly reduced. To this system we apply the quadrupole formula for the rotating rod. The numerical simulation of this system evolves in time intervals of about $10^{-22}$ s with an emission waveform (whose shape is very similar to the time reversed of that of in-spiraling binaries) with an abrupt change (very short time duration) after the fission completes. The emission power peaks when the rate of change of the quadrupole moment peaks. The quadrupole formula for the approximate GW power (quadrupole formalism), *P,* generated by a rotating rod with mass *M* and length *L* in free space is:

$$P = 2\mathrm{G}M^2L^4\omega^6/45\mathrm{c}^5 = 1.76\times10^{-52}\,(2\mathrm{r}\Delta f/\Delta t)^2\ \mathrm{W}. \qquad (1)$$

The derivation of the right side of Eq. (1) (Appendix of Baker, 2005) begins with the basic quadrupole approximation for GW power (Burdge, 2000) $P = (\mathrm{G}/45\mathrm{c}^5)(\mathrm{d}^3\mathrm{I}/\mathrm{dt}^3)^2\mathrm{W}$, where c is the speed of light and G the universal gravitational constant. We take the third-time-derivative of the moment-of-inertia, $\mathrm{I} = 2\delta\mathrm{m}\ \mathrm{r}^2$ (r, the radius of gyration = $L/2$ and $M = 2\ \delta\mathrm{m}$) of a dumbbell-shaped rod consisting of two masses, δm, at each end. We then apply Newton's-second-law, so that Eq. (1) is obtained. In Eq. (1) r is the radius-of-gyration and $\Delta f$ is the change in force at δm over the incremental time interval $\Delta t$ (that is, a "jerk"). Our simulation, based upon the quadrupole formalism, is expected to be accurate within a factor of ten. We choose a nucleus with atomic weight of 242 ($M$=4.04x$\cdot10^{-25}$kg), $L$= $10^{-16}$m and rotational energy of the isomer $\hbar\omega = .2$ MeV (Wiedenhover, 1999), equivalent to $\omega$=3.03x$10^{24}\mathrm{s}^{-1}$, we find $P_0$=2.85x$10^{-22}$W for each fission event. Supposing that 1 mg of fissionable material can be timed to emit pulses whose amplitude exactly superimpose at a given location, being generally $P=A^2/Z$, with $A$ amplitude and $Z$ the impedance of the medium. One milligram correspond to $N$=2.4x$10^{18}$ reactions, corresponding to a factor of $5.76 \times 10^{36}$ for the peak power, that therefore is found to be $P = 1.64 \times 10^{25}$ W. The practical implementation could be based on a rod shaped reactor with uniform internal distribution of fissionable atoms and plane wave fronts. Power could be uniformly distributed on a disk with diameter equal to the rod section with, for example, a shape ratio of 10 (length/diameter). We therefore have:

$$P \approx 4.8 P_0 \sqrt[3]{N^4} \quad . \tag{2}$$

In our example $P$ is equal to 910 W, which can increase up to the levels previously found depending on the optics of the fission inducing particles beam. The result is impressive and it is in part due to the direct summation of amplitudes, therefore it assumes linearity and no impedance matching. As soon as the impedance matching of the load (space) to the source is optimized the power tends to increase with the number of sources involved and all the rotational energy is extracted from them. An additional assumption is the fact that reactions should be timed with accuracy of the order of $10^{-22}$ s, which might require very sophisticated control mechanisms and geometries. Nevertheless it seems that nuclear fission and maybe nuclear fusion can take a very important role among the space technologies that will be developed for the generation of HFGWs. In fact, if nuclei have very high density, then they can be found in a rotational state and can be deformed in order to acquire a varying quadrupole moment, therefore becoming good emitters of HFGWs. We have described the phenomenon in form of a transient one-way effect, it might be possible to discover nuclei in which a cyclic behavior can be induced, but this is beyond the scope of this present paper.

If a *single pair* of the nuclear events occurs at the ends of an imaginary dumbbell or baton having a radius of gyration r, then the impulsive centrifugal force at each end will be:

$$\Delta f_{cf} = Mr\omega^2 = 3.7 \times 10^8 \text{ N} \tag{3}$$

over a time interval $\Delta t \sim 3.3 \times 10^{-22}$ s (Bonasera, Kondratyev and Iwamoto, 1997). Thus for the nuclear events occurring simultaneously and with parallel axes at exactly the same nuclear orientation some 20m apart (radius of gyration r = 10m), $P = 8.7 \times 10^{10}$ W. The engineering challenges of timing, paralleling, and orienting the nuclear events will be discussed. The wavelength is $c\Delta t = 10^{-13}$m. GW is generated, but $\lambda_{GW}$ is much less than 2r, and may lead to a poor power estimation using the quadrupole approximation of Eq. (1) as, for example, A. Pais (1982) and K. S. Thorne (1987), suggest. On the other hand, L.P. Grishchuk (2003) suggested that the requirement that $2r \ll \lambda_{sGW}$ may *not* be a stringent or even a necessary one for the quadrupole approximation to GW power to hold. As K. S. Thorne (1987) states "… the quadrupole formalism typically is accurate to within factors of order 2 even for sources with sizes of (the) order (of) a reduced (GW) wavelength …" Whether the quadrupole approximation to the power of gravitational wave generation holds or not does not necessarily imply that no GWs are generated by an impulsive force acting on a pair of masses or nuclei or that the power does not increase with the distance, 2r, between them. The quadrupole formalism may still provide order-of-magnitude estimates perhaps augmented by higher-order octupole, hexadecapole, etc. modes of pulsation and possibly reduced at the GW focus by diffraction. It is a problem deserving study in future. In any event considerable GW should be generated, but the approximations to GW power $P$ (from the quadrupole formalism) and amplitude A (Eqs. (1) and (3)) may not accurately hold especially when two or more GW radiators are many GW wavelengths apart.

## REACTOR DESIGN

The basic reactor design is based on the concept of traveling waves. If we have a cylinder filled with suitable fissionable material, then it is necessary to send fission-inducing particles along the main axis of the cylinder, and the speed of those particles should exactly match the speed of the traveling HFGW.

The basic structure of the reactor is schematized in Fig. 1. Dynamically, bunches of fission inducing particles enter from the left traveling at the speed of HFGW. While traveling along the reactor the fission inducing particles destabilize the spinning isomers converting rotational energy to gravitational waves, which then exit at the right. Fission fragments and neutrons that are normally the useful end products of nuclear reactors are actually unwanted and their effects should be minimized. This design is very similar to the one described in Baker (2000).

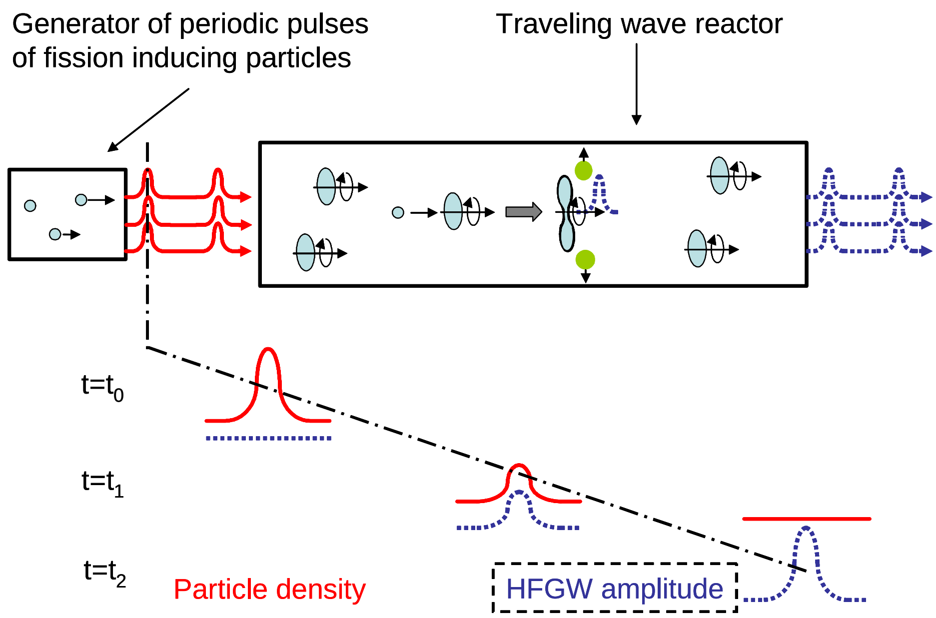


**FIGURE 1.** The Traveling Wave Reactor.

If fission inducing particles that can be practically produced travel at speed lower than the speed of HFGW, then they can be sent towards the cylinder at an angle. If this angle is 90 degrees, then they ideally induce fission simultaneously in the whole cylinder (if the diameter is negligible). If less than 90 degrees, then the fission progresses at an adjustable slower rate down the cylinder. The angled configuration is shown in Fig. 2. With the angled configuration the speed of the fission inducing particles is given by the speed of HFGW times the cosine of the angle between the two unitary vectors shown in the picture, which can match fission speed. To allow for a very precise timing of reactions, the reactor must not be self sustaining and should be sub critical. Fission inducing particles can be neutrons, they can be produced by small scale controlled fusion reactions (Naranjo, 2005), which is available technology.

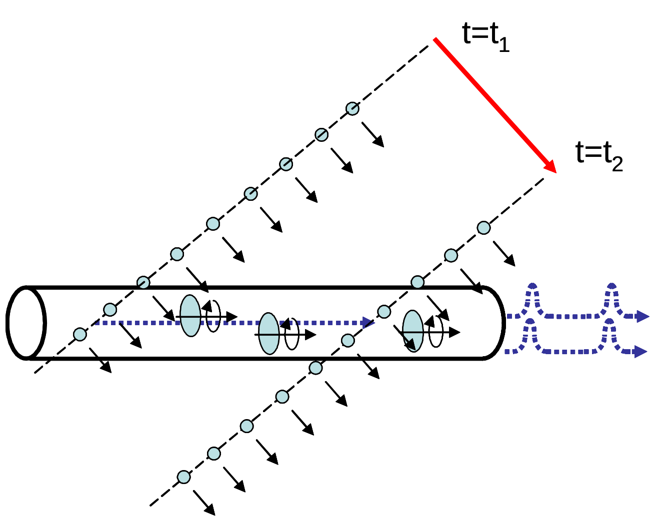


**FIGURE 2.** Nuclear Generator of HFGWs with slow fission inducing particles.

Unfortunately, once produced, neutrons are very difficult to control and to group into bunches. The ideal alternative is antiprotons (Baker, 2000). They can be produced, can be accumulated and can be controlled electromagnetically. It has been recently proposed (Kammash, 2005) that antiprotons are very efficient in inducing fission, therefore technical solutions are already available for possible experiments. The technology of the reactor can be assimilated to that of accelerator driven fission reactors, which are sub-critical and do not require moderators. Accelerator driven nuclear reactors are actually studied for incinerating nuclear waste (Petrov, 1992) (Krakowski, 1995). The design of Fig. 2 can be interpreted as mixed radiation optics, it could be possibly studied with optical simulation software. The wave of fission inducing particles, or actually its projection along the main axis of the reactor, must travel at the speed of light. Instead fission products coming from reactions already induced do not travel at the speed of light, they are slower, therefore the fission-inducing wave of particles is always in advance and the fission products from, say, the $2.4 \times 10^{18}$ reactions wouldn't disrupt the timing of these reactions along the cylinder. A single fission inducing particle generator could drive a number of reactors, producing various configurations for the output HFGW.

## SPACE TECHNOLOGY APPLICATIONS

Applications to space may lie primarily in the area nuclear effects related to space objects. Depending upon such an effect a novel nuclear-propulsion paradigm could be developed (Baker, 2005). Alternatively, a novel spacecraft power-generation scheme could be implemented. In order to estimate the largest possible effect for either of these applications, let us consider the largest possible generated HFGW power assuming the parameters and equations given in the nuclear fission section of this paper.

The largest power will be produced by a combination of the fission-inducing beams and the HFGW concentration at the focus between two such beams (that is, pairs of simultaneous, parallel reactions in proper sequence). As before the amplification caused by the coherent build up of N such timed reaction as the wave front advances is $P\mathrm{N}^2$. In the most optimistic case $P = 8.7\mathrm{x}10^{10}$ W and $\mathrm{N} = 2.4\mathrm{x}10^{18}$ reactions (for one mg of fissionable material). Thus $P = 5\mathrm{x}10^{47}$ W, but concentrated over an exceedingly small area. Specifically, the concentration extends over twice diffraction-limited spot area (since GW goes in both directions) having a radius of $\lambda_{GW}/\pi$, where $\lambda_{GW}$ is the wavelength of the HFGWs (Saleh and Teish (1991), p. 95, Eq. (3.2-17)). The HFGWs will probably have a large spread in wavelengths, but will be concentrated around $\lambda_{GW} = c\ \Delta t \sim 1.0\mathrm{x}10^{-13}$ m (which is a fraction of the diameter of a nucleus – for comparison, the classical radius of an electron is $3\mathrm{x}10^{-15}$ m and the first Bohr radius is $5\mathrm{x}10^{-11}$ m). Under these circumstances the area of concentration is $3\mathrm{x}10^{-27}\ \mathrm{m}^2$ and the HFGW flux is $F_{GW} \sim 1.6\mathrm{x}10^{74}\ \mathrm{Wm}^{-2}$. As derived in the Appendix A of Baker, Woods, and Li, 2006, the dimensionless GW amplitude (essentially the amplitude of a strain in spacetime, meters/meter) is

$$A = (8\pi G\, F_{GW}/c^3\omega^2)^{1/2} = 1.28\mathrm{x}10^{-18} F_{GW}^{1/2}/\nu_{GW}. \tag{4}$$

Equation (4) is strictly valid for monochromatic or quasi-monochromatic GW; but the GWs may cover a wide range of frequencies, the fundamental one being $\nu_{GW} = 1/\Delta t \sim 3\mathrm{x}10^{21}\ \mathrm{s}^{-1}$. Thus $A \sim 0.005$. Such a large GW amplitude should have significant effects on the interior of any nucleus that might be at the HFGW focus. Space-time singularities might be also generated with interesting applications to space travel (Fontana, 2005). A schematic of the apparatus for two beams composed for clarity of only two single lines of highly deformed and rapidly rotating nuclei "4" is shown in Fig. 3. The collective deformed nuclear model of extra particle(s) is shown in Fig. 5 of Hill and Wheeler (1953). As the HFGW-generation, short-time- duration reaction events move instep along the dual beams they produce HFGW radiation patterns "6" (Baker, Davis, and Woods, 2005) along the axis "5". The wave front that is thereby generated moves in the direction "7" and the reaction events are precisely timed so as to generate coherent and strong HFGW in the direction "2" and weaker incoherent HFGW in the opposite direction "3". The axes of the two beams are contained in the plain of the apparatus "8". The engineering challenge of timing, paralleling, and orienting the nuclear events might be met by spin polarization (Baker, 2000) of the dual beams and/or the beams passing through an intense electromagnetic or magnetic field (e.g., 15 T) and the nuclear reactions triggered in synchronization by ultra-high intensity lasers.

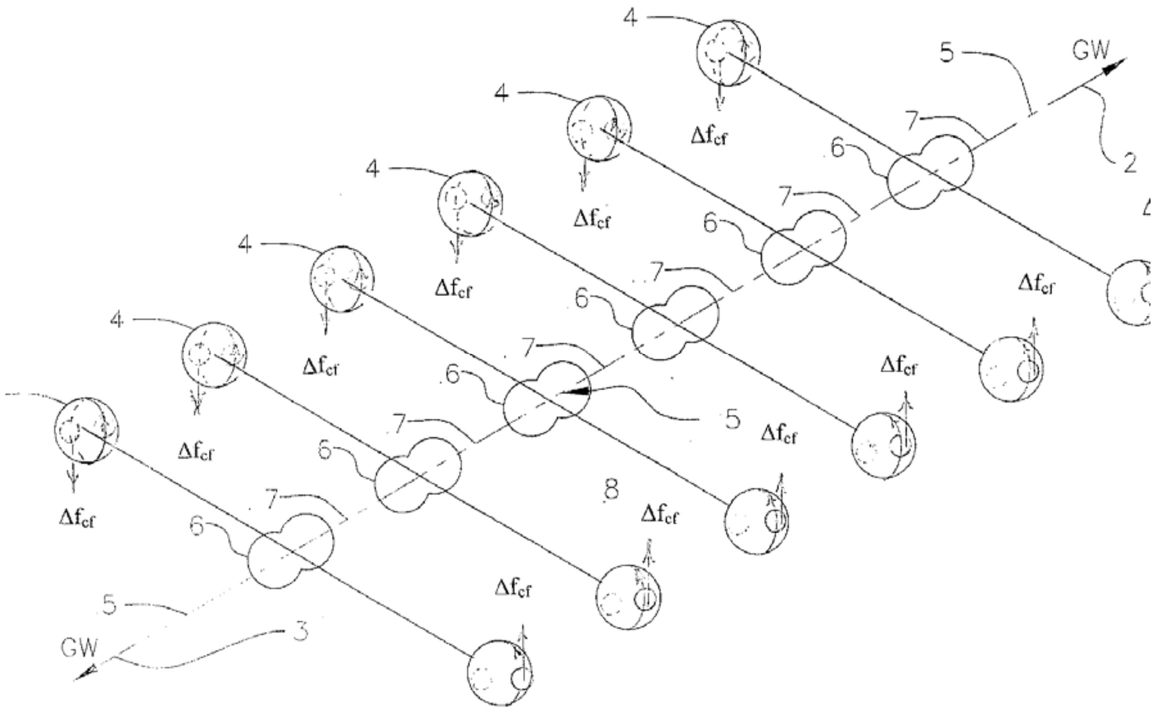


**FIGURE 3.** Schematic of the Dual-Beam, Nuclear-Reaction HFGW Generator.

Other applications could be similar to those described in Baker, Fangyu Li, and Ruxin Li, 2006 and in Baker, Woods, and Fangyu Li, 2006 in which the dual beams are placed extremely far apart at locations on the Moon and the stable $L_3$ lunar libration point. In this case the HFGW amplitude, A, would be increased to $(4x10^8/10)$ (0.005) = $2x10^5$ ($F_{GW}$ = $2.2x10^{89}$ $Wm^{-2}$ and for the focal-spot area of $3x10^{-27}$ $m^2$, $P$ = $7x10^{62}$ W) and very dramatic effects could be anticipated if the minute focal spot were in or near a nucleus. If we assume that a HFGW amplitude of one is the criteria for an extreme effect on matter, then from Eq. (4) $F_{GW}$ = $5.5x10^{78}$ $Wm^{-2}$. From Eq. (10) of Baker, Davis and Woods, 2005, the distance, D, from the focal spot for A = 1 would be about $4x10^{-8}$ m so that several nuclei might be affected. By very precisely varying timing and orientation of the reaction beams, the minute focal spot or source of HFGW could be located anywhere on or under the Earth's surface. The challenges of precise orientation and timing of the reaction events would still remain. If the amount of reactor material could be increased or other approaches to reactor design introduced to increase the reactions to a much larger number, then statistically some reaction times could overlap in time and the HFGW focus could be more accurately located.

## CONCLUSION

It is concluded that through the use of new technology, especially innovative reactor design, it may be possible to harness nuclear energy to generate ultra-high-intensity HFGWs. Bursts of such HFGWs could be concentrated in sub-nuclear volumes and would, no doubt, have a profound nuclear/gravitational effect that could even allow for a completely new view of nuclear physics and the fabric of spacetime.

It is also recommended that engineering technology, in the area of nuclear HFGW generation, be pursued in order to realize such expectations. A broad outline of the engineering configurations for such an advance has been presented in this paper.

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